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| Course Title: Real Analysis | Full Marks: 100 |
| Course No: Math Ed 423 | Pass Marks: 35 |
| Nature of Course: Theory | Period Per Week: 6 |
| Level: Bachelor Degree | Total Period: 150 |
| Year: Second |  |

**1. Course Description:** This course is designed to provide students with the fundamental concepts of Real Analysis. It deals with a rigorous development of the subject which includes the background of the knowledge of differential and integral calculus. The course focuses on the subject matter specially based on real line only. It covers foundation properties of real numbers, topological framework of real numbers, real sequences, infinite series of real numbers, limits, continuity, derivability and Riemann integrability of real functions.

**2. General Objectives**: The general objectives of this course are as follows:

* To acquaint the students with the axiomatic structure of real number system.
* To familiarize the students with the various properties of open and closed sets.
* To identify the features of real sequences and their convergence.
* To apply different tests for the convergence of infinite series.
* To develop a deeper understanding of the properties of infinite series in the students with the arbitrary terms and infinite product.
* To make students able to have a deeper understanding of limits of functions.
* To acquaint with the properties of continuous functions defined on an interval.
* To familiarize the students with the properties of uniform continuity, non uniform continuity of functions and monotonic functions.
* To enable the students in understanding the derivable and non-derivable functions at a point or on an interval.
* To apply the concept of derivative in expanding real functions in finite or infinite series.
* To make the students able to apply concept of derivative in examining extreme values of real functions.
* To familiarize the students with the Darboux sums of real functions on a closed interval.
* To acquaint the students with the properties of Riemann integrability of functions.

**3. Specific Objectives and Contents**

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| **Objectives** | **Contents** |
| * Describe different system of real numbers * Describe algebraic structure of real numbers. * Explain the properties of real numbers. * Describe order axioms of real numbers and their properties * Explain and prove properties of supremum and infimum of a set of numbers. * Explain completeness property Archimedian property and Dedekind property of real numbers. * Explain absolute value of a real number and its properties. | **Unit 1. Real Numbers (12)**  1.1 System of real numbers  1.2 Algebraic structure of **R**  1.3 Order axioms and properties of **R**  1.4 Absolute value of a real number and its properties.  1.5 Boundedness of subsets of **R**  1.6 Completeness axioms in **R**  1.7 Archimedian property  1.8 Dedekind's construction of the set of real numbers  1.9 Representation of real numbers on a line. |
| * Expkain neighbourhood of a point * Prove properties of open sets in **R** * Explain and prove properties of interior of a set. * Explain limit point of a set * Prove properties of limit points of a set and derived set * Prove properties of closed sets. * Prove properties of compact sets connected sets and perfect sets. | **Unit-II. Open and Closed Sets (15)**  2.1 Open and closed intervals  2.2 Neighbourhoods  2.3 Interior points and interior of a set  2.4 Open sets  2.5 Limit points of a set  2.6 Bolzano-Weirstrass's theorem  2.7 Closed sets  2.8 Covering of a set  2.9 Compact sets  2.10 Cantor sets  2.11 Connectedness |
| * Explain convergence of a sequence * Prove properties of convergent sequence of real numbers * Prove properties of a Cauchy sequence. * Explain non-convergent sequences and their properties * Prove properties of monotonic sequences * Describe subsequences and prove properties of subsequences * Explain uniform convergence of a sequence * Discuss the concept of Cesaro summability for sequences | **Unit-III Real Sequences (20)**  3.1 Convergent sequences  3.2 Cauchy Sequence  3.3 Cauchy's Criterion for convergence  3.4 Non-convergent sequences  3.5 Cauchy's first and second theorems on limit  3.6 Monotonic sequences, monotone convergence theorem, Cantor's intersection theorem  3.7 Subsequences  3.8 Uniform convergence  3.9 Summability of sequences |
| * Explain the conditions for the convergence of an infinite series * Establish different tests for the convergence of infinite series * Explain the convergence of an infinite series with positive and negative terms. * Explain absolute convergence and conditionally convergent of a series * Prove properties of the re-arrangement of the terms of an infinite series. * Prove the properties of the convergence of an infinite product * To discuss the Cesaro summability of series. | **Unit-IV. Infinite Series (25)**  4.1 Meaning of an infinite series  4.2 Sequence of partial sums of a series  4.3 Convergence of an infinite series  4.4 Cauchy's general principle of convergence of series  4.5 Series of positive terms  4.6 Different tests of convergence of series (comparison test, P-test, D-Alembert's,ratio test, root test, Rabee's test, Kummer's test, logarithmic ratio test)  4.7 Series of positive and negative terms  4.8 Alternative series and Leibnitz' test  4.9 Absolute convergent and conditionally convergent of series  4.10 Arbitrary series and infinite products, (Dirichlet's theorems), Abel's theorem)  4.11 Grouping of terms of a series  4.12 Re-arrangement of terms of series  4.13 Infinite product and its convergence  4.14 Summability of series |
| * Explain types of functions * Discuss boundedness of functions * Prove properties of limits of functions * Explain definition of limit of functions | **Unit-V Functions and Limits (10)**  5.1 Types of functions  5.2 Boundedness of function  5.3 Monotonic functions  5.4 Limit of functions  5.5 Properties of limits of functions  5.6 One sided infinite limits |
| * Explain the concept of continuity of a function at a point and on intervals * Prove theorems on algebra of continuous functions * Classify discontinuous functions at a point * Prove theorems on properties of continuous functions * Prove the theorems on properties of monotonic functions * Prove theorems on properties of uniform continuity of functions | **Unit-VI Continuity of Functions (18)**  6.1 Continuous functions  6.2 Continuity of intervals and sets  6.3 Properties of continuous functions  6.4 Discontinuous functions  6.5 Sign preserving theorem, intermediate value theorem, Bolzano theorem and fixed point theorem  6.6 Continuity of inverse functions  6.7 Continuity of monotomic functions  6.8 Uniform continuity of functions  6.9 Lipschitz functions |
| * Explain the concept of derivative * Establish the relation between continuity and derivability * Prove properties of derivability in relation to algebraic compostions * Prove Darboux theorem and its consequences * Establish and illustrate mean value theorems * Prove Taylor's series (finite and infinite) and Maclaurin's series * Discuss extreme values of functions * Discuss and prove various types of intermediate forms and their properties | **Unit-VII Derivability (22)**  7.1 Derivative of a function  7.2 Derivative of inverse function  7.3 Darboux theorem  7.4 Mean value theorems (Rolle's theorem, Lagrange's theorem, Cauchy's theorem)  7.5 Deductions from mean value theorems  7.6 Generalized mean value theorems  7.7 Taylor Polynomial  7.8 Power series representation of functions  7.9 Extreme values of a functions  7.9 Indeterminate forms  7.10 L-Hospital's rule. |
| * Establish relationship between lower and upper Darboux sums of a bounded function on a closed interval * Compute lower and upper Darboux sums of functions with a partition defined on the domain of definition of the function * Establish the properties of lower and upper Riemann integrals * Establish the necessary and sufficient condition for integrability of a function * Prove the properties of integrable functions * Establish mean value theorems of integral calculus * Establish fundamental theorem of integral calculus * Establish the techniques of evaluating the definite integrals, by integration by parts and change of variable of integration | **Unit-VIII Riemann Integral (28)**  8.Partitions  8.2 Lower and upper Darboux sums and properties of Darboux sums.  8.3 Upper and lower integrals  8.4 Riemann Integral  8.5 Necessary and sufficient condition for integrability  8.6 Properties of integrable functions  8.7 Mean value theorems (Mean value theorems of integral calculus, generalized mean value theorem)  8.8 Bonnet's and Weierstrass' mean value theorem.  8.9 Continuity and derivability of integrable functions  8.10 Fundamental theorems of integral calculus  8.11 Integration by parts  8.12 Change of variable |

**4. Instructional Techniques:** The nature of this course being theoretical, teacher centered teaching technique will dominate the teaching learning process. The teaching techniques are referred as follows:

**4.1 General instructional techniques**

* Lecture with illustrations
* Discussion
* Inquiry and question answer
* Demonstration

**4.2 Specific Instructional techniques**

* Individual and group work presentation for illustrations and exercise for all units

**5. Evaluation:** Students will be evaluated on the basis of written classroom test in between and at the end of academic session, the classroom participation, presentation of the reports, and other activities. The scores obtained will be used only for feedback purposes. The office of the Controller of Examination will conduct annual examination at the end of the academic session to evaluate students' performance. The types of questions and marks of each type of questions will be as follows:

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| **Types of questions** | **Total questions to be asked** | **No. of questions to be answered and marks to be allocated** | **Full Marks** |
| Multiple choice items | 20 questions | 201 marks | 20 |
| Short answer questions | 8 (with 3 alternatives) | 87 marks | 56 |
| Long answer questions | 2 (with 1 alternatives) | 212 marks | 24 |

**6. Recommended and Reference Books.**

**6.1 Recommended Books**

Gupta, S. L. & Gupta, N. R. (1993)*. Fundamentals of real analysis.* New Delhi, Vikas Publishing House Pvt. Ltd.

Maskey S.M. (2007), *Principles of real analysis* (second ed.). Kathmandu : Ratna Pustak Bhandar (For Units I-VIII).

Pandey U. N. (2003). *Real analysis* ( Fourth revised ed.2015 ) Kathmandu : Vidyarthi Prakashan Pvt. Ltd. ( For units I - VIII).

**6.2 References**

Apstol, T. M. (1997). *Mathematical analysis.* Tokyo: Addison Wesly Publishing Company

Bartle, R. G. & Sherbert, D. R. (1982). *Introduction to real analysis.* New York: John Wiley and Sons

Bhattarai, B. N. & Shrestha, B. K.,(2072). *A textbook of real analysis* (revised edition) Kathmandu: Shuvakamana Prakashan Pvt. Ltd.

Jain, P. K. & Kaushik S.K. (2001). *An introduction to real analysis,* New Delhi: S. Chand and Comp. Ltd.

Narayan, S. (1971). *A course of mathematical analysi.,* Delhi: S. Chand and Com. Ltd.

Rudin W. (194), *Principles of mathematical analysis,* Newyork; Mc. Graw Hill