Course title: Modern Algebra Full Marks: 100 Course No.: Math Ed. 433 Major Pass Marks: 35 Nature of the course: Theoretical Periods per weeks: 6

Level: B.Ed. III Year Total periods: 150 Time per period : 55 minutes

1. **Course Description**

This is an introductory course in modern algebra in mathematics education. It provides axiomatic foundation for further study of mathematics. The algebraic structures dealt in this course are groups, rings, fields and field extensions.

1. **General Objectives**

The general objectives of this course are as follows:

* To familiarize the students with the understanding of the basic algebraic structures.
* To develop capabilities among the students in proving theorems and problem solving techniques in algebra.
* To help them develop positive attitude towards modern algebra.
* To help them develop the knowledge of field extensions and to develop capabilities among the students in proving theorems of field extensions.

1. **Specific Objectives and Contents:**

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| **Specific Objectives** | **Contents** |
| * Define binary operation and explain algebraic structure. * Construct Cayley`s table for operation on a set. * Prove the properties of binary operation. * Decompose a set into equivalence classes and explain quotient structure. * Define groups and give example of groups. * Verify the laws of exponents in the product of integral power of elements of group. | **Unit I: Groups (25)**   * 1. Algebraic system   2. Operation, Cayley`s tables properties of binary operations.   3. Semigroups, monoids   4. Equivalence relation, quotient structures.   5. Definition and example of group   6. Integral power of elements of a group   7. Cyclic groups   8. Composition table   9. Elementary properties of groups and cyclic groups |
| * Prove some prosperities of integral power of elements. * Define cyclic group and give example of cyclic groups. * Test whether a given structure is a group or not. * Prove the elementary properties of groups and cyclic groups. * Define permutations and find all permutation of small sets. * Compute the products of permutation * Define permutation group and symmetric group Sn alternating group and dihedral group and prove the related theorems. * Define and give examples of group action and prove some related theorems. | * 1. Permutations and product of permutations   2. Group of permutations, symmetric group Sn, and dihedral group.   3. Group actions. |
| * Define subgroups and give examples of subgroups. * Define relation. * Find the groups with defining relations. * Prove the properties of subgroup. * Construct small subgroups. * Draw lattice diagram of subgroups. | **Unit II: Subgroups (15)**  2.1 Definitions and examples of subgroups.  2.2 Centralizer, normalizer  2.3 Properties of Subgroup5  2.4 Generators and defining relations.  2.5 Subgroups generated by subsets 5f a group  2.6 Lattice of subgroup. |
| * Define and construct co-sets. * Prove the simple properties of co-sets. * Define and construct quotient groups. * Define homomorphism and prove the simple properties of homomorphism. * Explain centralizer, normalizer, stabilizer, and orbits. * Prove Langrange`s theorem and calculate the order of products of subgroups. * Construct isomorphism of small groups. * Prove isomorphism theorems. * Define the direct products of groups and prove properties of internal and external direct products. | **Unit III: Normality, Co-sets, Quotient Groups and Homomorphism and Direct Products (25)**   * 1. Co-sets, quotients of groups, normality and homomorphism.   2. Algebra of subsets of group co-sets.   3. Properties of homomorphism.   4. Normalizer, stabilizer, centralizer orbits, Lagrange`s theorem.   5. Counting principle.   6. Isomorphism theorem: fundamental theorem, diamond and quotient isomorphism theorems and correspondence theorem auto-morphism.   7. Direct Products. |
| * Define and explain rings and give examples of rings. * Discuss the suitable types of rings with suitable examples. * Prove the properties of rings. * Define subrings, ideals and homomorphism of rings, extensions of rings. * Discuss different types of ideals, prime, maximal, nil-point and nil-ideas. * Prove the properties of subrings ideals, homomorphism and algebra of ideals and quotient rings. * Prove the properties of quotient ring by prime or maximal ideals * Prove the properties of direct product and direct sum of rings and ideals. * Explain the concept of factorization in integral domains. * Prove the properties of factorization domain. | **Unit IV: Rings, Subrings, Ideals and Homomorphisms (30)**  4.1. Definition and examples of rings and subrings.  4.2 Ideals and homomorphism  4.3 Algebra of ideals  4.4 Homomorphism of rings  4.5 Embedding and extension of rings.  4.6 Prime, maximal, nil-point and nil ideals.  4.7 Factorization domain.  4.8 Euclidean domain.  4.9 Direct products and direct sum of rings and ideals.  4.10 Principle ideal domain.  4.11 Unique factorization domain.  4.12 Properties of factorization domain  4.13 Ring of factorization |
| * Define polynomial rings and give examples of polynomials. * State the properties of polynomial rings and illustrate properties of polynomials with suitable example | **Unit V: Polynomial Rings (15)**  5.1 Definition and examples of polynomials division algorithm  5.2 Factorization of polynomials |
| * Define group actions and to prove the theorems on group actions. * Find the conjugate relations on a set. * Prove Cauchy`s theorems and Sylow`s theorems * Find the classification of finite groups. | **Unit VI: Sylow’s Theorems and Classi-fication of Finite Groups (15)**  6.1 Group actions on a set.  6.2 Conjugate relations.  6.3 Cauchy’s theorems  6.4 Sylow’s theorems  6.5 Classification of finite groups |
| * Define the extension of fields and prove the related theorem * Define irreducible polynomials and prove the related theorems * Find the adjunction of roots * Discuss algebraic extensions and prove the related theorems with examples * Define algebraically closed fields and prove related theorems * Define normal and separable extensions and prove their theorems * Define splitting fields and prove the related theorems with examples * Discuss and find multiple roots * Define finite fields and prove their theorems * State separable extensions and prove their theorems with examples | Unit **VII: Fields (25)**  7.1 Algebraic extent of fields  7.2 Irreducible polynomials and Eisenstein criteria  7.3 Adjunction of roots  7.4 Algebraic extension  7.5 Algebraic closed fields.  7.6 Normal and separable extensions  7.7 Splitting fields  7.8 Normal extensions  7.9 Multiple roots  7.10 Finite fields  7.11 Separable extensions |

1. **Instructional Techniques**

**4.1 General Instructional Strategies**

Because of the theoretical nature of the course, teacher-centered instructional techniques will be dominant in the teaching-learning process. The teacher will adopt the following techniques:

* Lecture with discussion
* Use of software (math lab, mathematical if possible)
* Investigative approach in problem solving

**4.2 Specific Instructional Strategies**

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| **Unit** | **Chapter** | **Instructional techniques** |
| I | Groups | Expository, Discussion and presentation |
| II | Subgroups | Expository, Discussion and presentation |
| III | Normality, co-sets, Quotient Groups and Homomorphism and Direct Products | Expository, Discussion and presentation |
| IV | Rings, subrings, ideals and Homomorphisms | Project work, Presentation |
| V | Polynomials Rings | Expository, Discussion and presentation |
| VI | Sylow’s theorem and classification of finite groups | Project work, Home assignment |
| VII | Fields | Class work, project work and assignment |

1. **Evaluation**

Students will be evaluated on the basis of the written classroom test in between and at the end of the academic session, the classroom participation, presentation of the reports and other practical activities. The scores obtained will be used only for feedback purposes. The Office of the Controller of the Examinations will conduct the annual examination at the end of the academic session to evaluate the student` performance. The types, numbers and marks of the subjective and objective questions will be as follows:

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| **Types of questions** | **Total questions to be asked** | **Number of questions to be answered and marks allocated** | **Total marks** |
| Group A: Multiple choice items | 20 questions | 20 × 1 marks | 20 |
| Group B: Short answers questions | 8 with 3 ‘or’ questions | 8 × 7 marks | 56 |
| Group C: Long answers questions | 2 with 1 ‘or’ questions | 2 × 12 marks | 24 |

1. **Recommended Books and References**

**Recommended books**

Bhattarai, B.N. (2011) *Introduction of Group Theory*, Kathmandu: Subhakamana Prakashan

Bhattarai, B.N. (2011) *Introduction of Rings and Modules,* Kathmandu: Subhakamana Prakashan

Dummit, D.S. & Foote R. (2002). *Abstract algebra*, New Delhi: Wiley India Reprint

Fraleigh, J.B. (2003). *A first course in abstract algebra*, India: Pearson Education Inc.

Herstine, I.N. (1986). *Abstract algebra*, New York: Macmillan Publishing Company

Koirala, S.P. & Bhattarai B.N. (2010) *A textbook on higher algebra*, Kathmandu: Pragya Prakashan

**References**

Durbin, J.R. (2005) *Modern algebra*, India: John Wiley and Sons Inc.

Hersteine, I.N. (2008) *Topics in algebra*, New Delhi: Wiley, India.

Maharjan, H.B. (2000) *First course in abstract algebra.* Kathmandu: Ratna Pustak Bhandar.

Maharjan, H.B. (2007) Group *theory,* Kathmandu: Bhundi Puran

Maharjan, H.B. (2008) Rings *and modules*, Kathmandu: Bhundi Puran

Shrestha, R.M. (2006) Elementary *linear algebra,* Kathmandu: Sukunda Pustak Bhawan

Stheth, I.H. (2002) *Abstract algebra,* New Delhi: Prentice Hall of India

Thomas, W.H. (1974) *Algebra,* New York: Springer Verlag Inc.